Question 14

(a) On graph paper, using a scale of 1 cm to 1 unit on each axis, draw the axes so that $0 \leq x \leq 16$ and $0 \leq y \leq 16$. [1]

L is the point (2,8). $\overrightarrow{LM} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$ and $\overrightarrow{LN} = \begin{pmatrix} 12 \\ -5 \end{pmatrix}$.

(b) Plot the point L and draw the vectors $\overrightarrow{LM}$ and $\overrightarrow{LN}$. [2]

(c) Write down $\overrightarrow{MN}$ as a column vector. [1]

(d) Calculate $|\overrightarrow{LN}|$, the magnitude of $\overrightarrow{LN}$. [2]

P is the midpoint of LM and Q is the midpoint of MN. G lies on LQ such that $LG : GQ = 2 : 1$.

(e) Write down as column vectors

(i) $\overrightarrow{LP}$, [1]

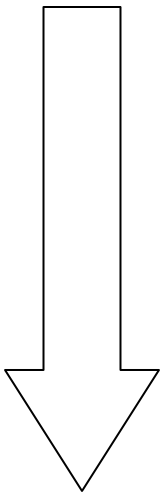
(ii) $\overrightarrow{NP}$, [1]

(iii) $\overrightarrow{LG}$. [1]

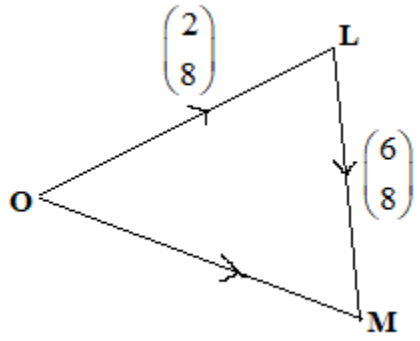
(f) Write down the coordinates of G. [1]

(g) Given that $NG = kNP$, calculate the value of k. [2]

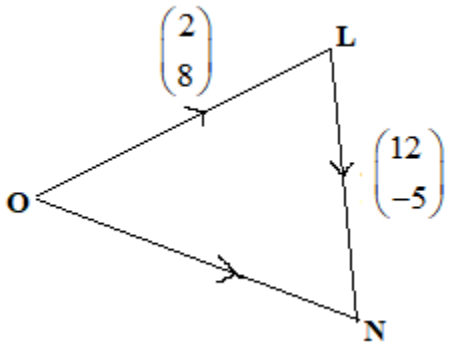
SOLUTION



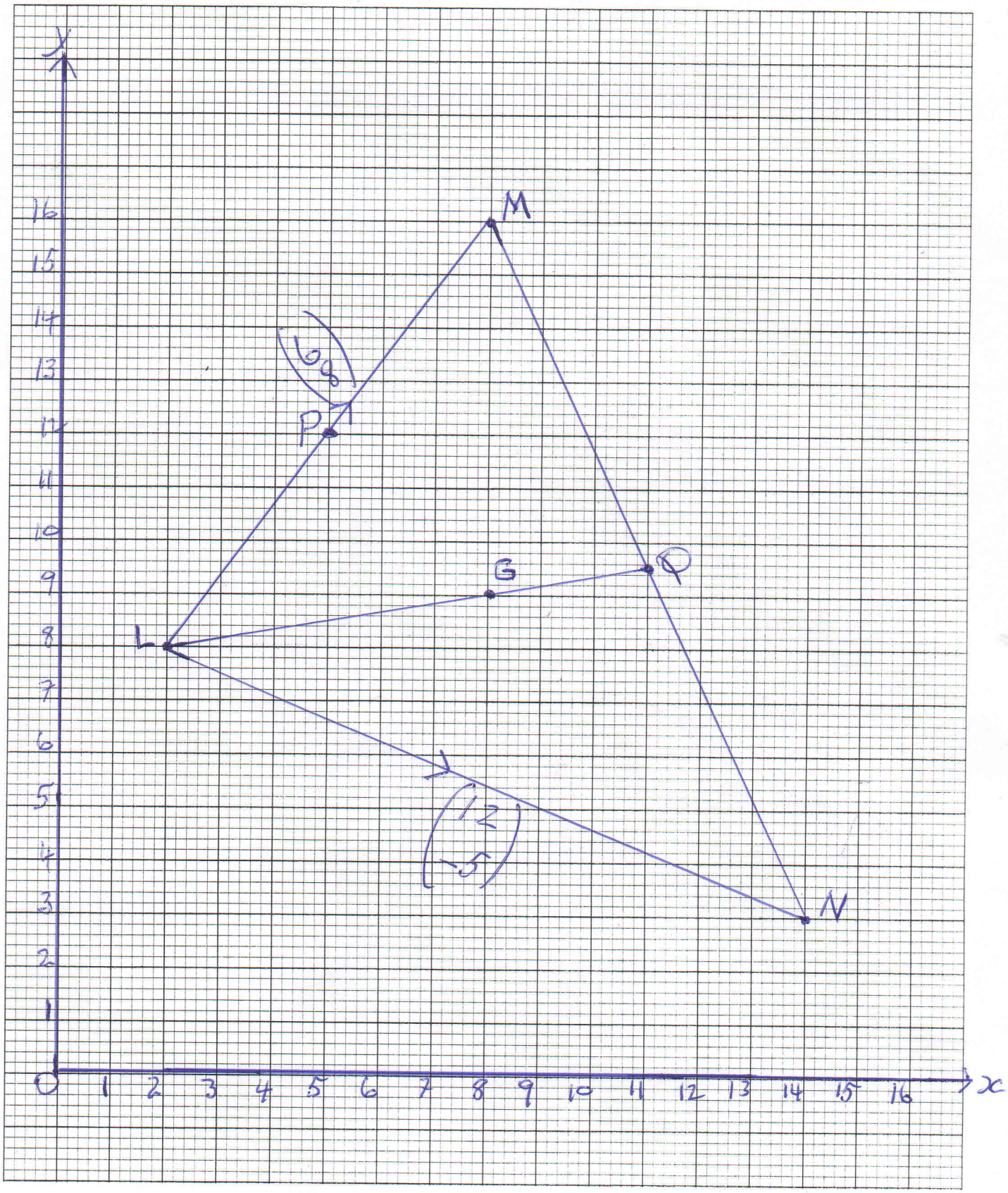
(b)



$$\overrightarrow{OM} = \overrightarrow{OL} + \overrightarrow{LM} = \begin{pmatrix}2\\8\end{pmatrix} + \begin{pmatrix}6\\8\end{pmatrix} = \begin{pmatrix}8\\16\end{pmatrix}$$



$$\overrightarrow{ON} = \overrightarrow{OL} + \overrightarrow{LN} = \begin{pmatrix}2\\8\end{pmatrix} + \begin{pmatrix}12\\-5\end{pmatrix} = \begin{pmatrix}14\\3\end{pmatrix}$$



y
x
O
M
P
Q
G
L
N
$\begin{pmatrix} 6 \\ 8 \end{pmatrix}$
$\begin{pmatrix} 12 \\ -5 \end{pmatrix}$

(c) $\overrightarrow{MN} = \overrightarrow{ML} + \overrightarrow{LN} = \begin{pmatrix} -6 \\ -8 \end{pmatrix} + \begin{pmatrix} 12 \\ -5 \end{pmatrix} = \begin{pmatrix} 6 \\ -13 \end{pmatrix}$

(d)

$$|LN| = \sqrt{12^2 + (-5)^2}$$
$$= \sqrt{144 + 25}$$
$$= \sqrt{169}$$
$$= 13$$

(e) (i) $\overrightarrow{LP} = \frac{1}{2}\overrightarrow{LM} = \frac{1}{2}\begin{pmatrix} 6 \\ 8 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

(ii) $\overrightarrow{NP} = \overrightarrow{NL} + \overrightarrow{LP} = \begin{pmatrix} -12 \\ 5 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} -9 \\ 9 \end{pmatrix}$

(iii) $\overrightarrow{LG} = \frac{2}{3}\overrightarrow{LQ} = \frac{2}{3}\left[\begin{pmatrix} 6 \\ 8 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 6 \\ -13 \end{pmatrix}\right] = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$

(f) Coordinates of G are (8, 9).

(g) $\overrightarrow{NG} = \overrightarrow{NL} + \overrightarrow{LG} = \begin{pmatrix} -12 \\ 5 \end{pmatrix} + \begin{pmatrix} 6 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 6 \end{pmatrix}$

$\begin{pmatrix} -6 \\ 6 \end{pmatrix} = k\begin{pmatrix} -9 \\ 9 \end{pmatrix}$, $k = \begin{pmatrix} -6 \\ 6 \end{pmatrix} \div \begin{pmatrix} -9 \\ 9 \end{pmatrix}$, $k = \frac{-6}{-9} = \frac{2}{3}$